J Therm Anal Calorim (2012) 107:387–393
DOI 10.1007/s10973-011-1469-x

Numerical simulation for the temperature changing rule of the crude oil in a storage tank based on the wavelet finite element method

Bin Zhao

Received: 12 February 2011 / Accepted: 9 March 2011 / Published online: 18 March 2011

Abstract In order to study the temperature changing rule of the crude oil in the storage tank, the wavelet finite element method, the traditional finite element method and the test were used to carry out the numerical simulation. Firstly, the thermal wavelet finite element was put forward established based on thermal finite element theory and the wavelet theory. And the computational model and three boundary conditions were established. And then the temperature changing rule of the crude oil in the storage tank in 24 h for three boundary conditions was obtained by using three methods, and the results showed that the wavelet finite element method had advantages in the numerical analysis of the temperature changing rule of the crude oil in the storage. And then the temperature distribution rule of the crude oil in the storage tank under different conditions in 5 h was obtained. And the temperature changing mechanism of the crude oil was summarized finally.

Keywords Temperature changing rule · Crude oil · Storage tank · Wavelet finite element method

Introduction

In order to fit in with the changing of the crude oil market, the projects of increasing the storage of the crude oil were executed in the oil field. There were a lot of large crude oil tanks in the oil field, as quantities of the crude oil tanks grown, which brought certain difficulty in the management of the oil storage. There were fixed times for crude oil in the storage tanks, the temperature of the crude oil could decreased as time prolonging, and the temperature of the crude oil should be kept above 302 K. Otherwise, the low temperature could bring the solidifying of the crude oil in the tanks. Therefore, it was necessary to carry out thermal-insulating measurement for the crude oil tanks, for example, the temperature-insulating coatings were installed on the outer wall of the crude oil tanks, which could decrease the loss of heat. In order to improve and optimize the heat-up scheme of the crude oil tanks, the temperature distribution rule of the crude oil in the storage tank should be studied in depth, the relation and its relationship with the ambient temperature, the inlet and outlet temperature of the crude oil tanks and the relation between the different status of the crude oil tank and the temperature of the crude oil should be got, and the main approach and the affecting factors of the heat loss of the crude oil tanks were found out.

The thermal behavior of the fluid in the storage tanks was considered by many scientists in recent years. A one-dimensional mathematical model of analyzing the temperature distribution of the liquid in the tank was put forward [1]. The changing rule of the temperature of the oil in the tank with the changing of the inlet temperature was studied in depth [2]. The transient thermal behavior of storage system based Chapeau-Galerkin integral formulation was investigated [3]. The effect of the tank geometry on the thermal behavior of the heat storage was analyzed [4]. The natural convention in the cylindrical enclosure was studied [5]. The cooling process of the fluid in the cylindrical tank by natural convention was analyzed [6]. The experimental analysis of the thermal performance of a hot water storage tank was carried out by [7].

The finite element technology was used in thermal calculation for the fluid in the storage tanks widely; however,

B. Zhao (✉)
School of Mechanical Engineering, Liaoning Shihua University, No. 1, West Dandong Road, Wanghua District, Fushun 113001, Liaoning Province, China
e-mail: zbzbz0203288@163.com

the traditional finite element method might come into being numerical oscillation or distortion during the process of solving the big gradient filed function. The wavelet finite element was the outcome combining the multi-resolution property of wavelet with the traditional finite element method, and using the wavelet function or scale function as interpolating function was a basic idea of the wavelet finite element method, and cover the shortage of traditional finite element method solving the big gradient and strange problems. Wavelet finite element application had attracted many researchers in a wide variety of practical problems. The temperature distribution was analyzed using wavelet finite element method, and the phenomenon of numerical distortion was avoided. The temperature distribution was analyzed using wavelet finite element method, and the phenomenon of numerical distortion was avoided [8]. The elastic problems of a plain plate with a circle hole were analyzed using wavelet finite element method [9], and the numerical results agreed with FEM. The rotor-bearing systems was analyzed by a wavelet finite element method, and the B-spline wavelet on the interval Rayleigh-Timoshenko shaft element considering the shear deformation was investigated, and the good performance of this method was demonstrated in the practical application and numerical examples [10]. All the mentioned numerical studies had carried out for the solid, and the study on the application of the wavelet finite element on the thermal performance of the fluid was very little.

The purpose of this research was to investigate numerically and experimentally, the thermal performance of the crude oil in the storage tank, and the changing rule of the temperature for the crude oil in the storage tank was simulated by using wavelet finite element, which could improve the study on the temperature changing rule of the crude oil in the tank. And the correctness of numerical results should be verified by comparing with the experimental results. The heat transfer mechanism of the crude oil in the storage tank could be got through analyzing the temperature change of the crude oil in the tank, and the intrinsic reasons would be found out. The results could offer the theory basis for inversing tank and heat preservation of the crude oil tank. And the solidify of the crude oil could be avoided, and the period of the inversing tank could be set properly, at the same time the energy wasted in the course of inversing tank could be saved, and the economical efficiency of the oil play could be improved.

Thermal finite element function

Daubechies wavelet scale functions $\varphi^1(\varepsilon)$ and $\varphi^2(\eta)$ generated multi-resolution sub-spaces $\left\{V_j^1\right\}$ and $\left\{V_j^2\right\}$, respectively, ε and η denoted the horizontal and vertical coordinate, respectively. And the tensor product of these three sub-spaces formed higher-level subspace [10]:

$$V_j = V_j^1 \otimes V_j^2 \tag{1}$$

where $\otimes$ was the Kronecker symbol.

$\left\{V_j\right\}$ generated a multi-resolution of the Hilbert space $L^2(R^2)$, which was defined by:

$$\vec{\varphi}^1 = \left\{\varphi^1(\varepsilon), \varphi^1(\varepsilon+1), \ldots, \varphi^1(\varepsilon+(N-2))\right\} \tag{2}$$

$$\vec{\varphi}^2 = \left\{\varphi^2(\eta), \varphi^2(\eta+1), \ldots, \varphi^2(\eta+(N-2))\right\} \tag{3}$$

The scale function in space $\left\{V_j\right\}$ was formulated by:

$$\vec{\varphi} = \vec{\varphi}^1 \otimes \vec{\varphi}^2 \tag{4}$$

The temperature field calculation of the diesel engine piston belonged to three-dimension problem, and the solution domain of temperature field could be discretized into finite elements, and the temperature of any nodes in element could be obtained using Daubechies wavelet scale function as interpolating function [11].

The temperature function $\theta(\varepsilon, \eta)$ could be formulated by [12]:

$$\theta(\varepsilon, \eta) = \vec{\varphi}\vec{a}^e = \sum_{i,j=0}^{N-2} \varphi_i(\varepsilon)\varphi_j(\eta)\vec{a}^e \tag{5}$$

where $\vec{a}^e$ was the corresponding vector of the wavelet filter coefficient, $\vec{a}^e = [a_0, \ldots, a_{-(N-2)}]^T$.

The heat transfer equation used in the process of temperature field calculation was

$$\frac{\partial^2\theta}{\partial\varepsilon^2} + \frac{\partial^2\theta}{\partial\eta^2} = 0 \tag{6}$$

$$-\left.\frac{\partial\theta}{\partial n}\right| = \alpha(\theta - \theta_f)|_s \tag{7}$$

where α denoted the heat transfer coefficient, W/(m^2 K); θ_f denoted the temperature of medium, K, which could be obtained through test and calculation; n was a normal vector of the boundary.

The corresponding functional equation could be expressed as follows:

$$\Pi(\theta) = \int_S \left\{\frac{\lambda}{2}\left[\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2 \mathrm{d}x\mathrm{d}y\right]\right\} + \int_\Gamma \left(\theta^2 - \theta_f\theta\right)\mathrm{d}\Gamma \tag{8}$$

where λ indicated thermal conductivity, W/(m K); V was the volume of the diesel engine piston, S was a computational region, Γ was a boundary.

In order to make the wavelet element consistent with $\vec{a}^e$, the allocation of nodes for the hexahedron was founded.

$\theta_{i,j}$ was the temperature of the node (i,j), the relations between local coordinate (ε,η) and global coordinate (x,y) was as follows [13]:

$$\varepsilon = \frac{x - x_1}{x_2 - x_1} \quad (9a)$$

$$\eta = \frac{y - y_1}{y_2 - y_1} \quad (9b)$$

where x_1, x_2, y_1, y_2 were minimum and maximum coordinate value of the element in different direction, respectively.

The functional equation could be expressed by formula (10):

$$\Pi^e(\theta) = \int_{se} \left\{ \frac{\lambda}{2}\left[\left(\frac{\partial\theta}{\partial\varepsilon}\right)^2 + \left(\frac{\partial\theta}{\partial\eta}\right)^2 + \right]\right\} \mathrm{d}\varepsilon\mathrm{d}\eta + \int_{\Gamma e} \left(\theta^2 - \theta_f\theta\right)\mathrm{d}s \quad (10)$$

where e indicated the element.

The differential equation of three dimensions steady thermal transfer could be got by the condition $\delta\Pi^e(\theta) = 0$ of the functional equation mentioned above [14].

$$\iint_{se} \lambda\left(\frac{\partial W \partial\theta}{\partial\varepsilon\,\partial\varepsilon} + \frac{\partial W \partial\theta}{\partial\eta\,\partial\eta}\right)\mathrm{d}\varepsilon\mathrm{d}\eta + \oint_{\Gamma} (2\theta - \theta_f)\frac{\partial\theta}{\partial n}\mathrm{d}\Gamma = 0 \quad (11)$$

where W denoted the weight function, θ_f was a reference temperature, choosing interpolating function as weight function could obtain [15]:

$$\vec{N}^e \vec{a}^e + \vec{K}^e \vec{a}^e = \vec{P}^e \quad (12)$$

where $\vec{a}^e$ denoted the differential coefficient column vector of unknown wavelet coefficients, $\vec{N}^e$, $\vec{K}^e$, $\vec{P}^e$ denoted the heat capacity matrix of the element, heat exchange matrix of the element, heat load matrix of element, respectively. $\vec{N}^e$, $\vec{K}^e$, $\vec{P}^e$ could be expressed as follows [16]:

$$\vec{N}^e_{m,n,l;i} = \int_0^1\int_0^1 \lambda \Phi_{m,n}\Phi_{i,j}\mathrm{d}\varepsilon\mathrm{d}\eta \quad (13)$$

$$\vec{K}^e_{m,n,l;i} = \int_0^1\int_0^1 \lambda\left(\frac{\partial\Phi_{m,n}\partial\Phi_{i,j}}{\partial\varepsilon\quad\partial\varepsilon} + \frac{\partial\Phi_{m,n}\partial\Phi_{i,j}}{\partial\eta\quad\partial\eta}\right)\mathrm{d}\varepsilon\mathrm{d}\eta + \oint_{\Gamma e} \Phi_{m,n}\Phi_{i,j}\mathrm{d}s \quad (14)$$

$$\vec{P}^e_{m,n} = \oint_{\Gamma e} \left(\lambda\Phi_{m,n} + \Phi_{m,n}\theta_f\right)\mathrm{d}\Gamma \quad (15)$$

The global differential equation of thermal conduction could be established after the element matrix superimposed and the boundary conditions was handled

$$\vec{N}\vec{\theta} + \vec{K}\vec{\theta} = \vec{P} \quad (16)$$

where $\vec{N}$, $\vec{K}$, $\vec{P}$ denoted the whole heat capacity matrix, the whole heat exchange matrix, and the whole heat load matrix, respectively

The difference of the time was carried out by Crank-Nicolson format [17]:

$$\frac{1}{2}\left(\left.\frac{\partial\theta}{\partial t}\right|_t + \left.\frac{\partial\theta}{\partial t}\right|_{t-\Delta t}\right) = \frac{\left(\vec{\theta}_t - \vec{\theta}_{t-\Delta t}\right)}{\Delta t} + O\left(\Delta t^2\right) \quad (17)$$

where Δt denoted the time step, $O(\Delta t^2)$ denoted the truncation error. The simultaneous equation of formula (15) and formula (16) in t and $t - \Delta t$ was [17]:

$$\left(\vec{K} + 2\vec{N}/\Delta t\right)\vec{\theta}_t = \left(\vec{P}_t + \vec{P}_{t-\Delta t}\right) + \left(2\vec{N}/\Delta t - \vec{K}\right)\vec{\theta}_{t-\Delta t} \quad (18)$$

The temperature field calculation of the diesel engine piston could be calculated depending on formula (18).

The computational model and boundary conditions

After the crude oil entered the storage tank and the status of the crude oil was stable, the crude oil in the tanks would begin to cool because the ambient temperature was lower than the temperature of the crude oil. The external environment condition of the tank could be confirmed firstly, and then original physical properties of the crude oil should be confirmed, and then the boundary conditions and the original conditions of the mathematical model were confirmed. The two-dimensional geometric model of the crude oil in the tank was shown in figure [5].

The physical significance of the symbol in Fig. 1 was introduced as follows: T_a, temperature of the external environment; T_1, temperature of the crude oil; T_l

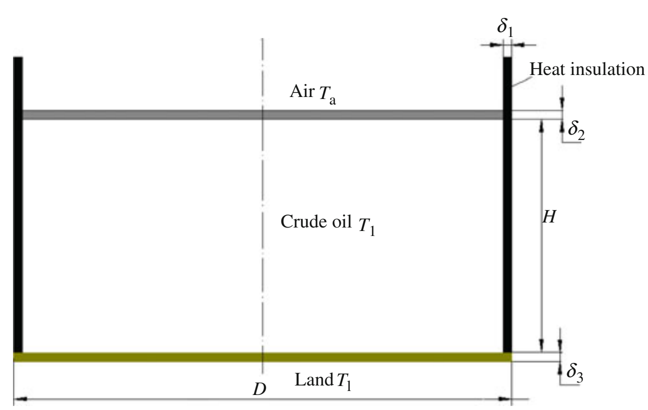

Fig. 1 The two-dimensional geometric model of the crude oil in the tank

temperature of the land; δ_1, thickness of the heat insulation ($\delta_1 = 60$ mm); δ_2, thickness of the tank roof ($\delta_2 = 5$ mm); δ_3 the thickness of the tank base ($\delta_3 = 10$ mm); H the liquid level of the crude oil in the tank; and D the diameter of the tank ($D = 6000$ mm).

The specific heat of the crude oil was 2340 J/(K kg), and the conductivity of the crude oil was 0.23 W/(m K), and the viscosity of the crude oil was 28.2×10^{-6} m^2/s. The tank roof and tank base were manufactured by steel plate, and the specific heat of the steel plate was 465 J/(K kg), and the density of the steel plate was 7900 kg/m^3, and the conductivity of the steel plate was 36.7 W/(m K). The specific heat of the heat insulation was 754 J/(K kg), the density of the heat insulation was 75 kg/m^3, and the conductivity of the heat insulation was 0.035 W/(m K). The flow velocity of the air above the tank roof was 0.1 m/s.

Three boundary conditions were considered in this research. The boundary conditions of the three boundary conditions were listed as follows:

First boundary conditions of the crude oil tank: the liquid level of the crude oil in the tank was 5 m, and the original temperature of the crude oil was 42.5 °C, and the mean wind speed was 2.6 m/s.

Second boundary conditions of the crude oil tank: the liquid level of the crude oil in the tank was 10 m, and the original temperature of the crude oil was 45.8 °C, and the mean wind speed was 2.1 m/s.

Third boundary conditions of the crude oil tank: the liquid level of the crude oil in the tank was 17.5 m, and the original temperature of the crude oil was 46.3 °C, and the mean wind speed was 4.5 m/s.

The liquid level of the crude oil in the tank was constant for different boundaries, and the changing curve of the external environment in this research was shown in Fig. 2 [3].

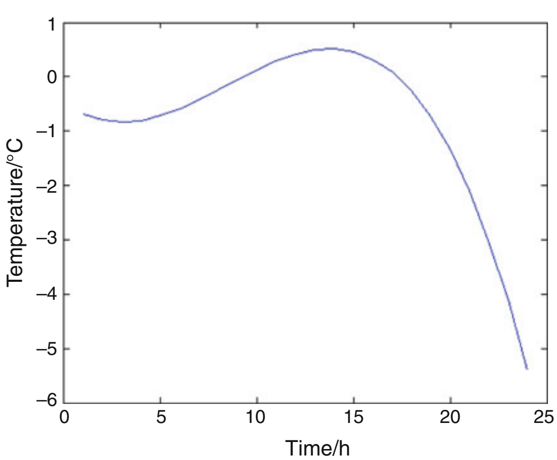

Fig. 2 The changing curve of the external environment

Numerical analysis of the temperature changing rule of the crude oil in the storage tank based on wavelet finite element method

Research methods

The temperature changing rule of the crude oil in the storage tank was analyzed based on the wavelet finite element method. In order to verify the advantages of the finite element method, the temperature changing rule of the crude oil in the storage tank was analyzed by traditional finite element method and test, respectively. The wavelet finite element analysis program was compiled by MATLAB software, the meshing was established by Daubechies wavelet finite thermal element and the PLANE77 in ANSYS software, respectively. PLANE77 has one degree of freedom, temperature, at each node, and it was well suited to thermal analysis. The temperature transducer was used to test the temperature of the different location of the crude oil in the tank [10].

For the first boundary conditions of the computational model, the crude oil in the tank was meshed by 614 wavelet finite element and 15,524 finite elements in ANSYS software. And the temperature transducer was fixed on the location where the liquid level of the crude oil in the tank was 5 m.

For the second boundary conditions of the computational model, the crude oil in the tank was meshed by 823 wavelet finite element and 19,539 finite elements in ANSYS software. And the temperature transducer was fixed on the location where the liquid level of the crude oil in the tank was 10 m.

For the third boundary conditions of the computational model, the crude oil in the tank was meshed by 967 wavelet finite element and 22,396 finite elements in ANSYS software. And the temperature transducer was fixed on the location where the liquid level of the crude oil in the tank was 17.5 m.

Computational results

The changing curves of the temperature of the crude oil in the storage tank in 5 h were shown in Figs. 3, 4, and 5. It could be seen from the graph that the temperature changing speed of the crude oil in the storage tank varied directly with difference between the original temperature of the crude oil and the ambient temperature, and was inversely proportional to the liquid level of the crude oil in the storage tank approximately. With the increasing of the difference between the original temperature and the ambient temperature the temperature changing speed of the crude oil in the tank would speed up. And with the increasing of the liquid level of the crude oil in the tank

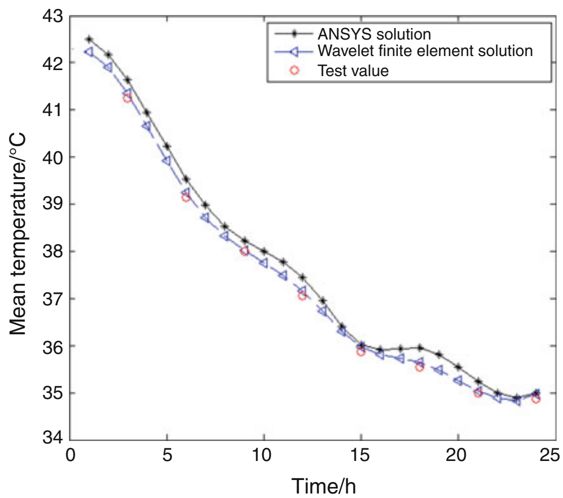

Fig. 3 The temperature changing curves of the crude oil in the tank for the first boundary condition

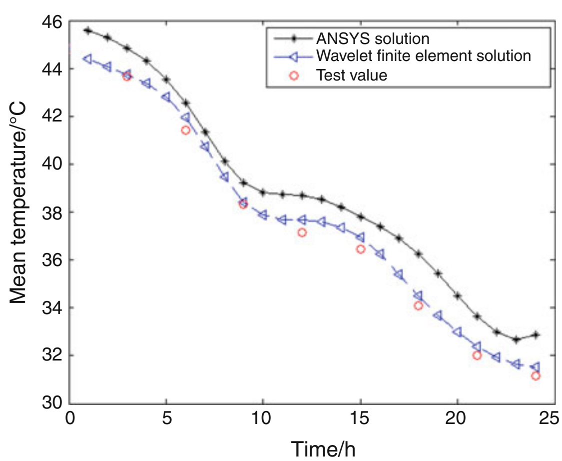

Fig. 4 The temperature changing curves of the crude oil in the tank for the second boundary condition

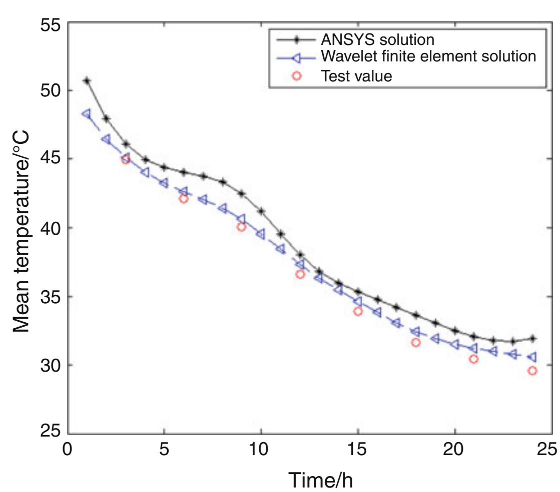

Fig. 5 The temperature changing curves of the crude oil in the tank for the third boundary condition

the temperature changing speed of the crude oil in the tank would decrease [14].

The solutions got from ANSYS software and wavelet finite element and the values got from actual test were given in the same figure. It could be seen from the graph that the wavelet finite element solution was nearer to the test value than the ANSYS solution, which illustrated that the wavelet finite element method had a higher precious than the traditional finite element. At the same time, the wavelet finite element method could use less wavelet finite elements to get higher precious than the traditional finite element method, which showed that the finite element had the advantages in analyzing the thermal behavior of the crude oil in the storage tank.

The temperature changing isothermal diagrams of the crude oil in the storage tank in 5 h for three boundary conditions were shown in the Figs. 6, 7, and 8. In the course of the temperature changing of the crude oil in the storage tank, regardless of the high and low of the liquid level of the crude oil, the high temperature zone of the crude oil in the storage tank was centered largely in the upper middle section of the crude oil tank. The temperature of the top part of the crude oil tank was higher than the bottom of the tank along the liquid level of the crude oil tank.

The high temperature zone of the crude oil was not centered in the center of the crude oil tank, while the

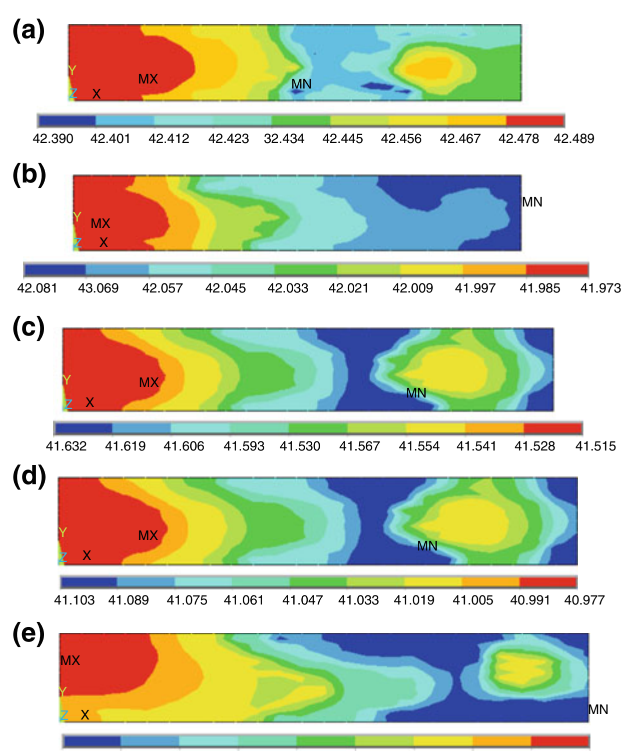

Fig. 6 The temperature distribution of the crude oil in the tank for the first boundary conditions after **a** 1 h, **b** 2 h, **c** 3 h, **d** 4 h, **e** 5 h

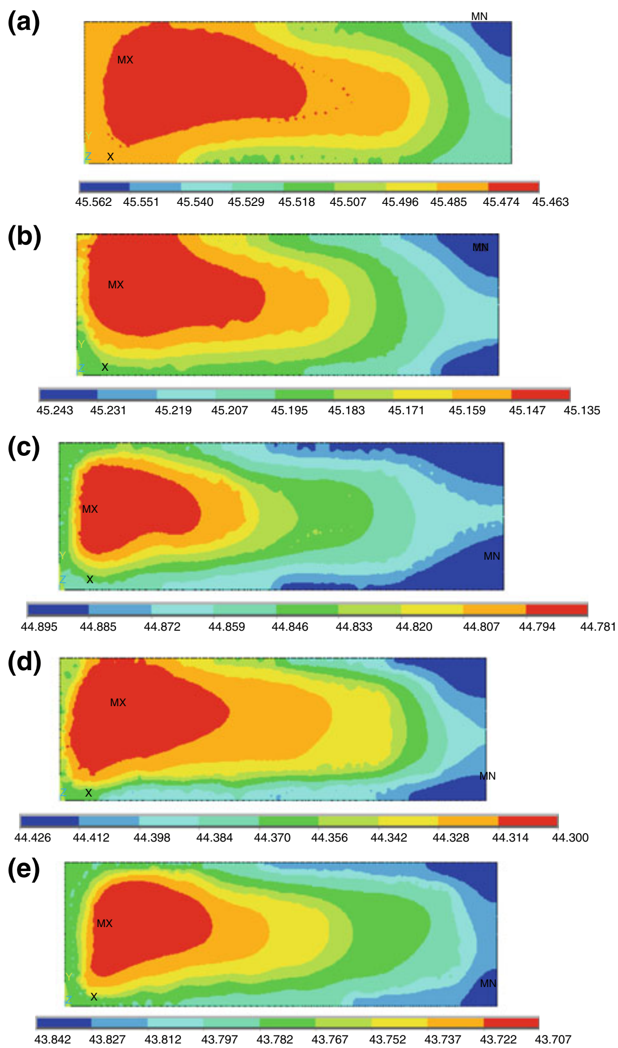

Fig. 7 The temperature distribution of the crude oil in the tank for the second boundary conditions after **a** 1 h, **b** 2 h, **c** 3 h, **d** 4 h, **e** 5 h

temperature of the crude oil near to the tank wall was highest, the main causes of this phenomenon were: There were not heat insulation on the floating roof of the crude oil tank and the bottom of the crude oil tank, therefore the fast natural temperature changing of the crude oil in the storage tank was formed. However, there were heat insulations on the crude oil tank wall, therefore the temperature changing speed of the crude oil was low.

The temperature changing speed of the crude oil in the storage tank was fastest in the initial period of the storage crude oil stage. In the early of the storage crude oil stage, the difference between the original temperature and the ambient temperature was biggest, and the thickness of the crude oil reservoir was lest, and the heat resistance was lest, and the heat radiation fluctuation was biggest, and the

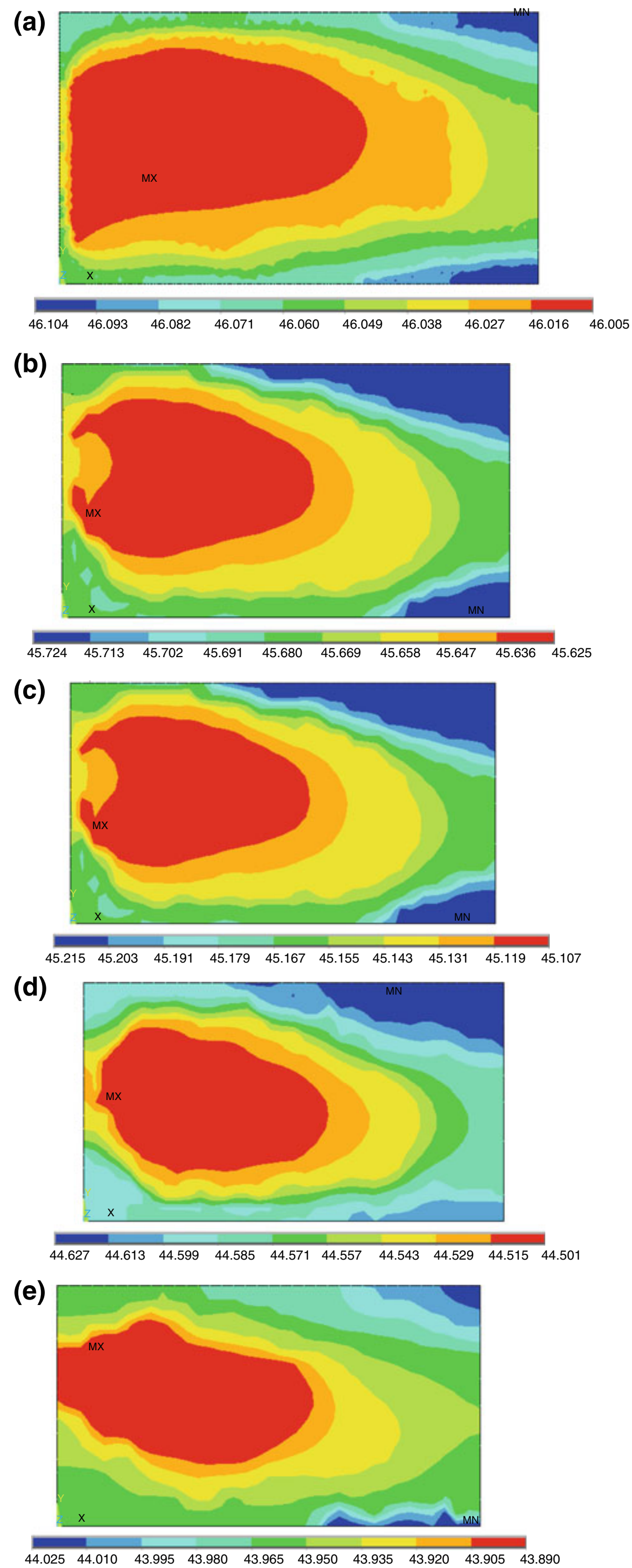

Fig. 8 The temperature distribution of the crude oil in the tank for the third boundary conditions after **a** 1 h, **b** 2 h, **c** 3 h, **d** 4 h, **e** 5 h

viscosity of the crude oil was relative lowest, the conventional intensity of the crude oil was biggest, and the temperature of the crude oil distributed uniformly. After the rapid temperature changing of the crude oil, the crude oil could solidify, and the mean temperature of the crude oil could decrease, and with the increase of the temperature changing time, the temperature changing speed of the crude oil could slow. The reasons of rapid temperature changing of the crude oil were: (1) with the increase of the crude oil reservoir the heat resistance improved; (2) a lot of heat loss was offset by the latent heat got from the condensation of the crude oil.

Conclusions

The temperature changing rule of the crude oil in the storage tank was analyzed by wavelet finite element method, traditional finite element method and actual test. The wavelet finite element for studying the thermal performance of the crude oil was established by combining Daubechies wavelet theory and the traditional finite element method. The temperature changing rules of the crude oil in the storage tank in 24 h were obtained by using numerical and experimental method, respectively. It could be seen from the graph that the temperature changing speed of the crude oil in the storage tank varied directly with difference between the original temperature of the crude oil and the ambient temperature, and was inversely proportional to the liquid level of the crude oil in the storage tank approximately. The temperature changing isothermal diagrams of the crude oil in the storage tank in 5 h for three boundary conditions were obtained. The high temperature zone of the crude oil was not centered in the center of the crude oil tank, while the temperature of the crude oil near to the tank wall was highest. The relationship between the temperature changing speed of the crude oil in the storage tank and the temperature changing time was analyzed finally. The temperature changing speed of the crude oil in the storage tank was fastest in the initial period of the storage crude oil stage. The advantages of using wavelet finite element method for thermal problem compared to the conventional FEM by analyzing the calculating results. The wavelet finite element had higher precious and shorter calculating time than the traditional finite element method.

Acknowledgements This research was supported by Liaoning education department scientific and technical research project (L2010244).

References

1. Gari HN, Loehrke RI. A controlled buoyant jet for enhancing stratification in a liquid storage tank. J Fluids Eng. 1982;104: 475–81.
2. Abu Hamdan MG, Zurigat YG, Ghajar AJ. An experimental study of a stratified thermal storage under variable inlet temperature for different inlet design. Int J Heat Mass Transfer. 1992;35:1927–33.
3. Al-Najem NM, El-Refaee MM. A numerical study for the prediction of turbulent mixing factor in thermal storage tanks. Appl Therm Eng. 1997;17:1173–81.
4. Eames PC, Norton B. The effect of tank geometry on thermally stratified sensible heat storage subject to low Reynolds number flows. Int J Heat Mass Transfer. 1998;41:2131–42.
5. Papanicolaou E, Belessiotis V. Transient natural convection in a cylindrical enclosure at high Rayleigh numbers. Int J Heat Mass Transfer. 2002;45:1425–44.
6. Lin W, Armfield SW. Direct simulation of natural convection cooling in a vertical circular cylinder. Int J Heat Mass Transfer. 1999;42:4117–30.
7. Manral L, Gupta PK, Suryanarayana MVS, Ganesan K, Malhotra RC. Thermal behaviour of fentanyl and its analogues during flash pyrolysis. J Therm Anal Calorim. 2009;96:531–4.
8. Al-Najem NM, El-Refaee MM. A numerical study for the prediction of turbulent mixing factor in thermal storage tanks. Appl Thermal Eng. 1997;17:1173–81.
9. Fernandez-Seara J, Uhia Francisco J, Sieres J. Experimental analysis of a domestic electric hot water storage tank. Part I: static mode of operation. Appl Therm Eng. 2007;27:129–36.
10. Chen X-f, Yang S-j, Ma J-x, et al. The construction of wavelet-finite element and its application. Finite Elem Anal Des. 2004; 40:541–54.
11. Suárez AC, Tancredi N, Pinheiro PCC, Yoshida MII. Thermal analysis of the combustion of charcoals from *Eucalyptus dunnii* obtained at different pyrolysis temperatures. J Therm Anal Calorim. 2010;100:1051–4.
12. Ramezanzadeh B, Mohseni M, Yari H, Sabbaghian S. A study of thermal–mechanical properties of an automotive coating exposed to natural and simulated bird droppings. J Therm Anal Calorim. 2010;102:13–22.
13. Díaz LA, Martín MT, Vampa V. Daubechies wavelet beam and plate finite elements. Finite Elem Anal Des. 2008;45:200–9.
14. Amaratunga K, Sudarshan R. Multiresolution modeling with operator-customized wavelets derived from finite elements. Comput Method Appl Mech Eng. 2006;195:2509–32.
15. Korneev V, Langer U, Xanthis L. On fast domain decomposition methods solving procedures for hp-discretizations of 3d elliptic problems. Comput Method Appl Mech Eng. 2003;3:536–59.
16. Doğan F, Ulusoy M, Öztürk ÖF, Kaya İ, Salih B. Synthesis, characterization and thermal study of some tetradentate Schiff base transition metal complexes. J Therm Anal Calorim. 2009;98: 785–92.
17. Felix FS, Cides da Silva LC, Angnes L, Matos JR. Thermal behavior study and decomposition kinetics of salbutamol under isothermal and non-isothermal conditions. J Therm Anal Calorim. 2009;95:877–80.